Effects of Firing Angle Imbalance on 12-pulse Rectifiers with Interphase Transformers

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***Abstract*-Firing angle imbalance between six-pulse groups of a twelve-pulse rectifier leads to current imbalance, due to the finite magnetizing inductance of the interphase transformer. The amount of current imbalance is limited by the load regulation of the six-pulse groups. This paper presents a quantitative analysis of this effect and others, based on appropriate models of the system. Equations are developed which predict steady-state current shifts and transient behavior using an averaged model. A piecewise-linear model of the system is used to verify and extend the results via computer simulation. Results are normalized, making them useful for design purposes.**

INTRODUCTION

Fig. 1 shows a 12-pulse rectifier composed of two 6-pulse bridges paralleled through an interphase transformer (IPT). The two three-phase source sets that supply the 6-pulse bridges are phase displaced by 30° with respect to each other, while the IPT is designed to force a continuous dc current in each bridge by absorbing the instantaneous difference between the output voltages of the bridges. As long as there is no average difference between bridge voltages, the IPT will function properly. In the presence of an average voltage, however, the IPT will saturate, causing the 12-pulse circuit to operate with each device conducting for only half of the previous conduction angle, while carrying twice as much current. This problem is especially acute in phase-controlled converters, where an average difference can easily arise from firing angle imbalances. This paper analyzes the current imbalance problem, demonstrates that the negative feedback implicit in a converter with load regulation, i.e., commutating impedance, provides corrective action, and quantifies the extent of this correction.

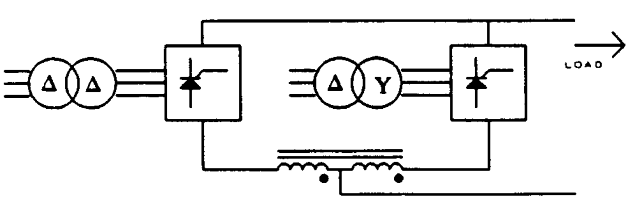


Fig. 1 A twelve pulse rectifier with interphase transformer.

Unbalanced operation of the 6-pulse bridges in a 12-pulse converter has been studied by others. Bennell described the dependence of current balance on mismatch and coupling between line transformer secondaries [1], while Evans has analyzed the interaction between supply harmonics and converter load balance [2]. Subbarao, *et. al.* considered the problem of firing angle imbalance in 12-pulse rectifiers configured as series-connected 6-pulse bridges, but inherent to this configuration is the independence of bridges and absence of an interphase transformer [3]. Others have investigated the effects of firing angle imbalance on line transformer operation [4,5]. However, these analyses do not consider the presence of commutating reactance and thus are not applicable to the general case. The only work to have addressed the problem of firing angle imbalance in phase-controlled converters with interphase transformers is that of Tanaka, *et. al.* [6], who present a piecewise model for simulating the current balance between converters, and use it to analyze some specific examples. A feedback control system for balancing bridge currents is also described. However, because their conclusions are based on some particular examples, they are not applicable to the general case.

This paper presents a quantitative analysis of the effects of firing angle imbalance on the current distribution in 12-pulse converters with interphase transformers, based on models that accurately represent the dynamic behavior of the system. A negative feedback effect is identified, in which the load regulation of each bridge has a stabilizing effect on the current balance between bridges. A new averaged model is derived which directly predicts the effects of a firing angle imbalance. The effects of unbalanced turns ratios and commutating reactances are also analyzed. A piecewise linear model of the system is then used to verify the results and extend them to cases where averaging is not justified. Both the analytic and simulation-based results are presented in a normalized form, making them valuable for design or analysis purposes.

CIRCUIT MODEL

As a first step, we propose a model for analyzing the system of Fig. 1, shown in Fig. 2. If the IPT were ideal, it would force the load current to be shared equally between the two parallel bridges. However, the magnetizing current of the IPT has significant impact on circuit behavior, and therefore the magnetizing inductance L_μ must be included in the model. Load regulation in the six-pulse bridges is the source of the feedback which stabilizes the current balance. Thus, input inductances L_c are included as part of the model, representing reactance of transformers, line filters, or line impedance. Finally, as in [6], we assume a constant current load, ideal switches, and negligible winding resistance in the transformers. This model can be used to simulate the detailed behavior of the circuit, but does not yield analytic expressions for the current balance. To achieve this, we turn to averaged modeling.

CIRCUIT BEHAVIOR

Averaged modeling

We are interested in finding an expression for the difference in average output current of the two six-pulse bridges. We will consider an approximation to the system in which each bridge carries a local average current and delivers a local average output voltage, as determined by its load regulation characteristic. The local average is defined as a moving average over a finite period of operation [7]. This will lead to a tractable solution which is exact in the limiting case of an infinite magnetizing inductance.

Consider the behavior of the circuit of Fig. 2 under a slight differential in the firing angles of the two bridges. We will assume that bridge 2 firing signals lag bridge 1 signals by $\Delta\alpha$, with α denoting the firing angle of bridge 1. The firing angle differential generates a dc component in the voltage across the IPT. This causes the magnitude of the magnetizing current i_μ to increase with time, forcing the load to be shared unequally by the bridges, with the higher voltage bridge carrying more load current. As the unbalance increases, the load regulation produced by L_c will cause the average output voltage of the higher voltage bridge to decrease, and that of the lower voltage bridge to increase. At some point (presuming the IPT does not saturate), the voltage difference caused by the firing angle differential will be completely compensated by the effect of load regulation. There will then be no dc component of voltage across the IPT, and a steady-state current imbalance will have been established.

We now examine the averaged behavior more formally. We will work under the assumption that the magnetizing current ripple is small (i.e., that the magnetizing inductance L_μ is large), allowing the magnetizing current i_μ to be replaced by its local average value $\bar{i}_\mu$. For our purposes, the local average of a variable will be defined as follows:

$$\bar{x}(t) = \frac{3\omega}{\pi} \int_{t-\frac{\pi}{3\omega}}^{t} x(\tau)d\tau. \tag{1}$$

where $\pi/(3\omega)$ is the ripple period of the bridges. The results of relaxing the constraint on ripple in i_μ will be considered in a later section. Under the additional constraint that $\Delta\alpha$ varies more slowly than one half the switching frequency $(3\omega/\pi)$, we can use circuit averaging to predict the local-average dynamics of the current balance [7]. In an averaged sense, a bridge with load regulation has the same terminal characteristics as an appropriate voltage source with series resistance [8]. Representing the bridges this way leads to the averaged model of Fig. 3, in which V_s is the peak line-to-line

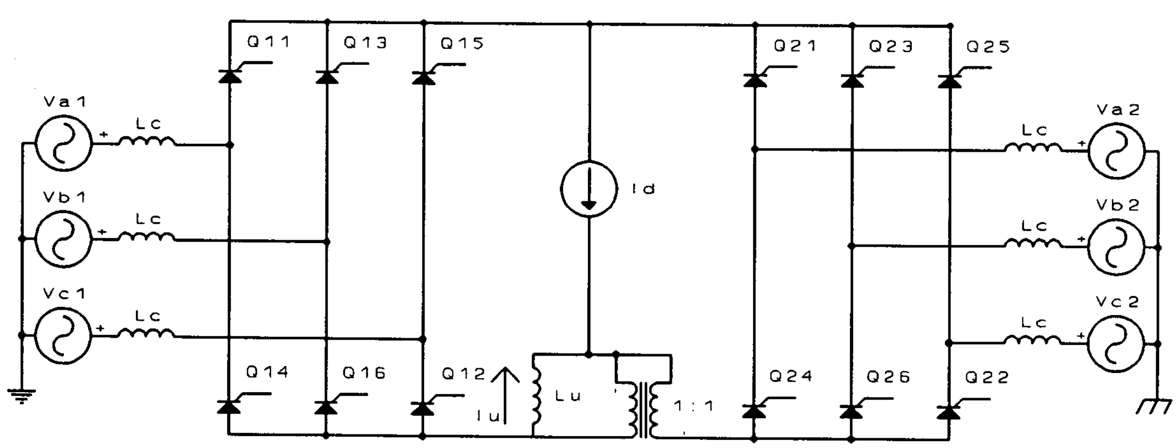


Fig. 2 A circuit model for the 12-pulse rectifier.

ac voltage.

An analysis of the circuit model of Fig. 3 shows that the system will respond to an initial firing angle differential of $\Delta\alpha$ with an L/R time constant of:

$$\tau = \frac{2\pi L_\mu}{3\omega L_c}. \tag{2}$$

This is apparent by inspection when the transformation of Fig. 4 is used to make the circuit symmetric. A slowly varying $\Delta\alpha$ will cause current shifts with the same first-order dynamic behavior. Solving for the (average) magnetizing current resulting from a constant $\Delta\alpha$ and normalizing the equation in terms of the total output current I_d yields:

$$\frac{\bar{i}_\mu}{I_d} = \frac{-2}{\left(\frac{X_c I_d}{V_s}\right)} \sin\left(\alpha + \frac{\Delta\alpha}{2}\right) \sin\left(\frac{\Delta\alpha}{2}\right). \tag{3}$$

Here we have expressed the relative current shift in terms of the firing angle α, the firing angle differential $\Delta\alpha$, and the reactance factor $X_c I_d / V_s$. The implication of this equation is that (ignoring ripple and saturation) when $\bar{i}_\mu / I_d = 1$, the entire load current is carried by one bridge, since the local average bridge currents are $\bar{i}_{d1} = (I_d - \bar{i}_\mu)/2$ and $\bar{i}_{d2} = (I_d + \bar{i}_\mu)/2$. Thus, we have an analytic expression for approximating the average load current imbalance that can be expected in a system under a constant firing angle imbalance.

Using the averaged model to analyze the effects on output voltage of the unbalanced currents predicted by (3), we find an approximation to the average output voltage of the twelve-pulse rectifier under firing angle imbalance:

$$\langle v_d \rangle = \frac{3V_s}{\pi}\left[\frac{\cos(\alpha + \Delta\alpha) + \cos(\alpha)}{2} - \frac{X_c I_d}{2V_s}\right]. \tag{4}$$

This equation is similar to the expression for $\langle V_d \rangle$ under balanced conditions, i.e.,

$$\langle v_d \rangle = \frac{3V_s}{\pi}\left[\cos(\alpha) - \frac{X_c I_d}{2V_s}\right]. \tag{5}$$

The only difference is that $\cos(\alpha)$ is replaced by the average of $\cos(\alpha)$ and $\cos(\alpha + \Delta\alpha)$.

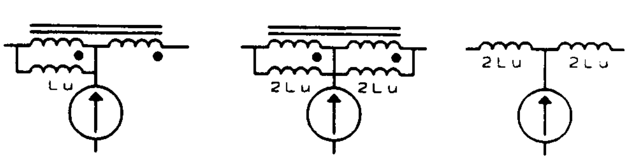


Fig. 4 Equivalent IPT models under constant output current.

Simple variations on the model of Fig. 3 can be used to predict the effects of various other conditions on current balance. For example, because the desired turns ratios of the transformers cannot be met exactly, there is often a voltage unbalance between the transformer secondaries of the two bridges [1]. Using imbalanced input voltages in the model, we find an average magnetizing current results for balanced firing angles ($\Delta\alpha$):

$$\frac{\bar{i}_\mu}{I_d} = \frac{(k-1)\cos(\alpha)}{\left(\frac{X_c I_d}{V_s}\right)} \tag{6}$$

where k is the ratio of the bridge 2 to bridge 1 secondary voltages.

Similarly, if the firing angles and turns ratios are correct, but commutating inductances of the two bridges are not equal, a steady state magnetizing current is predicted:

$$\frac{\bar{i}_\mu}{I_d} = \frac{L_{c1} - L_{c2}}{L_{c1} + L_{c2}}. \tag{7}$$

This result agrees with the observations of Bennell [1] for the modeled case of line transformer secondaries with no mutual coupling.

Note that the bridge currents that result from these equations are also correct if individual output inductors are used for current sharing, as long as they are of sufficient size to make the assumption of small current ripple valid. As shown in the transformations of Fig. 4, the two cases are equivalent if the output current I_d is constant, and $L_s = 2L_\mu$.

Piecewise modeling

While the averaged analysis yields a convenient analytic expression for the current shift, it works under the approximation that all variables are accurately represented by their local averages. To capture circuit behavior more precisely and to determine the limits of the averaged model, we turn to time domain simulation of the system using piecewise linear modeling.

Treating the thyristor switches as ideal, we can analyze the circuit behavior in terms of the circuit response during individual switch configurations, with each switch configuration being a set of switch states (on or off). Associated with each switch configuration is a set of state equations and a set of validity conditions. The state equations describe the trajectory of the system during a single switch configuration, while the validity conditions describe when the switch configuration becomes invalid.

The procedure for simulating the system is to calculate the

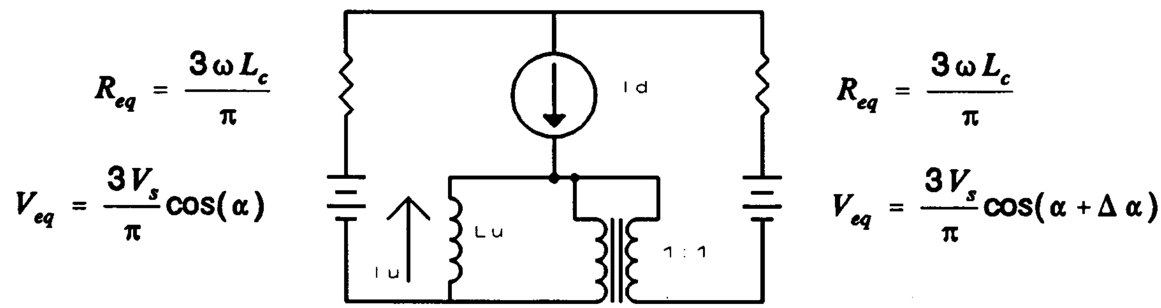


Fig. 3 An averaged model for the twelve-pulse rectifier.

state trajectory within each switch configuration, while numerically solving for the transition boundary of the next switch configuration. Because the state equations take on a simple form in this system, analytic expressions can be obtained for the state trajectories in each switch configuration. This eliminates the need for numerical integration, resulting in a computational advantage over other simulation methods which require it, such as that used in [9].

This approach is very similar to the one taken in [6], with a few basic differences. The equations of [6] are a reduced set of the state trajectory and validity equations used here. Because only global changes in bridge output current are tracked, the method used in [6] is simple and compact. However, the fact that it only tracks one variable limits its usefulness. Because the piecewise model we use tracks *all* of the inputs, state variables, and boundary conditions, it can be used to investigate a variety of conditions, such as unbalanced voltages or reactances, which the reduced model in [6] cannot.

A computer program has been constructed to simulate the system in this manner, with full control over system parameters, firing angles, etc. In addition to constructing the detailed current waveforms in the system, the simulator uses a trapezoidal integration method to calculate the average magnetizing current during steady state operation. (Note that the integrator is only used to calculate the average current, and not the state trajectory.) This simulation method provides a more precise view of circuit operation than does averaged modeling.

RESULTS

In order to make the data useful in a general context, it is necessary to normalize it. The dimensioned parameters of this system are ω, L_c, L_μ, $\mathcal{I}_\mu$, V_s, and I_d. Equation (3) suggests that two appropriate normalized parameters are the relative current shift $\mathcal{I}_\mu/I_d$, and the reactance factor X_cI_d/V_s. The remaining independent parameter is the magnetizing inductance, L_μ. L_c and L_μ appear as a ratio in (4), implying that $X_\mu I_d/V_s$ should also be a valid dimensionless parameter. The validity of normalizing the results this way has been extensively verified via direct simulation. By representing data in this manner, the results are made independent of specific parameters and are useful in the most general context.

Piecewise model simulations verified the averaged model predictions in all cases where the approximation of small ripple applied. Fig. 5 shows both piecewise and averaged predictions of steady state current balance for different values of $\Delta\alpha$, while Fig. 6 shows the steady state current shifts predicted by the two models for input voltage unbalance between bridges. Fig. 7 compares the responses predicted by the two models for a step change in $\Delta\alpha$. As long as $\Delta\alpha$ varies on a time scale longer than that of the waveform ripple, the simple first order model should predict the averaged current dynamics accurately.

One assumption used in the averaged model is that the magnetizing inductance is large enough to limit the effects of the current ripple in each bridge. Fig. 8 shows the results of

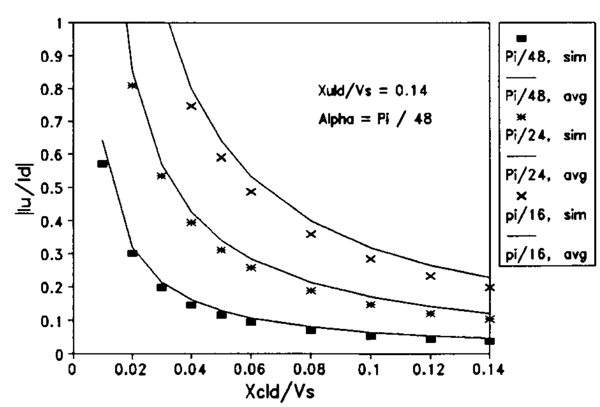


Fig. 5 Piecewise and averaged current balance predictions for three $\Delta\alpha$'s.

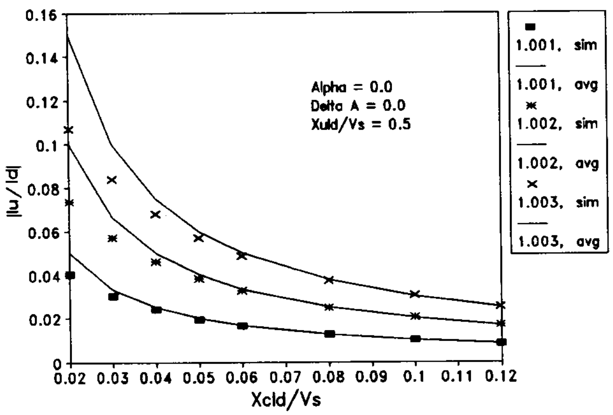


Fig. 6 Piecewise and averaged current balance predictions for three k's.

varying the size of the magnetizing reactance and thus the ripple in the bridge currents for two firing angles. As can be seen, the averaged model accurately predicts the current shift as the magnetizing reactance factor $X_\mu I_d/V_s$ becomes large. However, as the magnetizing reactance factor becomes small, significant deviations from this approximation can result, especially at small values of α. Thus, the conclusion of [6] that the size of the interphase reactance does not affect the average current shift is only valid under certain conditions. In the unusual case where the magnetizing reactance is of the same order as the commutating reactance (or smaller), normalized simulation data should be used instead of results from an averaged model.

Errors in predicting the average current shift using an averaged approach are due to the model used for load regulation in a six-pulse bridge, which is based on a constant output current. For finite magnetizing inductance, interactions between the two bridges cause a ripple in i_μ. The

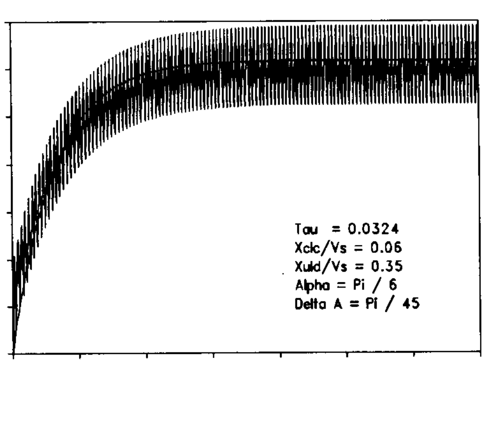


Fig. 7 Piecewise and averaged responses to a step in $\Delta\alpha$.

ripple current affects the commutation times and hence the load regulation of the bridges, resulting in the prediction error. This effect is in some respects similar to the converter interactions analyzed by Freris for *ac*-side coupling [10].

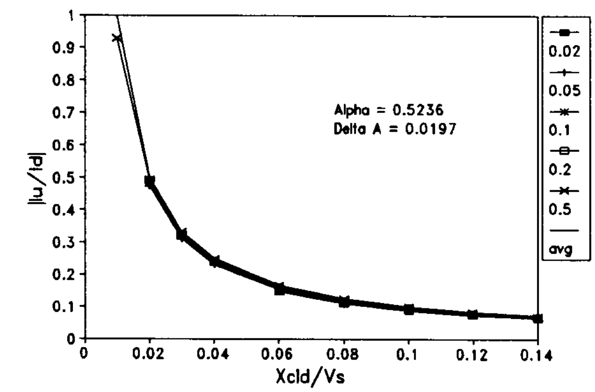
Another effect which needs to be addressed is the possibility of mode changes in the converter. Under conditions of large ripple and/or current shift, one of the bridges may enter a discontinuous conduction mode. This is the reason some of the simulation curves are not extended down to low values of $X_c I_d/V_s$. Analytic prediction of this effect is complicated, due to the interaction between bridges. Thus, when conditions of low magnetizing reactance factor and/or large current shifts are encountered, piecewise linear modeling should be used to analyze the system. An effect which may occur under heavy load conditions is the transition of converter operation from mode 3 (single commutations in each bridge, nonoverlapping) to mode 4 (single commutations, overlapping) [9]. Mode 4 is entered when the commutation period of one of the bridges exceeds $\pi/6$. While the precise onset of this transition is difficult to predict, a rough estimation can be made by using the commutation angle of each bridge assuming constant output currents. For a six pulse bridge, the commutation angle is:

$$\mu = \cos^{-1}\left(\cos(\alpha) - \frac{2X_c I_{dc}}{V_s}\right) - \alpha \tag{8}$$

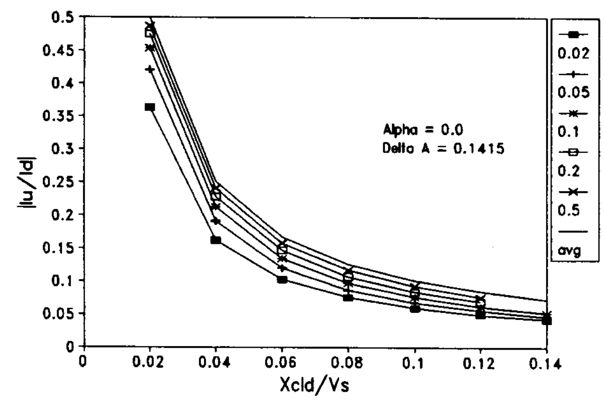
Also, as long as the magnetizing inductance is sufficiently large to limit the interactions between converters, (3) is valid for both modes 3 and 4. It remains valid until the commutation period of one of the bridges exceeds $\pi/3$.

Another observation which can be drawn from Fig. 8 is that a much smaller $\Delta\alpha$ is needed to cause the same current shift as α is increased, with the most drastic reductions in $\Delta\alpha$ occurring at small α. This effect is also apparent from (3), where an increase in α from zero amplifies the effect of $\Delta\alpha$ rapidly at first, then less as the sinusoid flattens out. What may be concluded from these observations is that given a system where there is an inherent minimum $\Delta\alpha$ possible in control, the effects of this imbalance must be evaluated at the maximum α to be used (out to $\alpha = 90°$). Furthermore, it is clear that in a system where α varies significantly from zero, the feedback due to load regulation is of limited usefulness in constraining current imbalances.

It has been demonstrated that feedback control based on current imbalance, as shown in [6], or core flux, as shown in [11], can be effective for balancing currents in interphase transformers. The averaged model of Fig. 3 (or a variant including the load dynamics) is a useful tool for designing the feedback control, since it describes the averaged dynamics of the system. Under the constraint that the bandwidth of the control system is far lower than the switching frequency,



(A)



(B)

Fig. 8 Effects of varying magnetizing reactance factor.

control designs based on this model should yield the desired results. The piecewise model can be used to verify the performance of the closed loop system.

CONCLUSION

The effects of firing angle imbalance between the six-pulse groups of a twelve pulse rectifier have been examined. It has been shown that the amount of current imbalance is limited by the load regulation of the six-pulse groups. Equations have been developed to calculate the dynamics of the current balance using an averaged model. The effects of unbalanced turns ratios and commutating reactances have also been analyzed. These results have been verified via computer simulation using a piecewise model of the system.

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