# Variable Frequency Multiplier Technique for High Efficiency Conversion Over a Wide Operating Range

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Abstract—This paper presents a Variable Frequency Multiplier (VFX) technique that enables design of converters for wide input and/or output voltage ranges while preserving high efficiency. The technique is applied to an LLC converter to demonstrate its effectiveness for converters having wide input voltage variation such as universal input power supplies. This technique compresses the effective operating range required of a resonant converter by switching the inverter and/or rectifier operation between processing energy at a fundamental frequency and one or more harmonic frequencies. The implemented converter operates over an input voltage range of 85 V to 340 V but the resonant tank and conversion ratio has only been designed for half this range; a VFX mode of the inverter is used to enhance this to the full range. Experimental results from a 50 W converter show an efficiency of 94.9% to 96.6% across the entire input voltage range, demonstrating the advantage of using this technique in such applications.

#### I. INTRODUCTION

A trend in power electronics has been to strive for high power density and high efficiency across a wide operating range [1]. High power density can be achieved by switching power converters at a high frequency. At these high frequencies, resonant converters use soft switching (i.e. Zero Voltage Switching (ZVS) and/or Zero Current Switching (ZCS)) to reduce switching losses to achieve high efficiency [2], [3]. Although soft-switched resonant converters can achieve high efficiency at a nominal operating point, the efficiency tends to degrade considerably with variations in input voltage, output voltage and power level [4].

Resonant converters commonly use frequency control [2], [3] and/or phase shift control [5], [6] to compensate for variations in input voltage and power levels. If switching frequency is increased to reduce output power or gain of the converter, such as in a series resonant converter operated above resonance to maintain ZVS, switching losses increase. Also, with operation over a wide frequency range as often required in resonant converters, the magnetics

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cannot be optimally designed. Furthermore, circulating currents may increase proportionally as load is decreased resulting in higher losses at light loads. With phase shift control, operation over a wide range is likewise challenging. In many resonant converters, when two legs of the inverter are phase shifted with respect to each other, they have asymmetrical current levels at the switching transitions. The leading inverter leg can lose ZCS and the lagging leg can lose ZVS. Other control techniques such as asymmetrical current mode control [7] and asymmetrical duty cycle PWM control [8] also have limitations such as loss of ZVS.

In this paper, a Variable Frequency Multiplier (VFX) technique is introduced and its effectiveness is demonstrated for a universal input power supply. In the VFX technique, additional "frequency multiplier" modes of operation of the inverter and/or rectifier are used to provide additional sets of operating characteristics for the converter to achieve and maintain high performance across a wide operating range. Frequency multiplier circuits are often used in extreme high-frequency RF applications (e.g., where transistor  $f_T$  is a concern), and are sometimes used in switched-mode inverters and power amplifiers (e.g., [9], [10]). While it has been proposed to employ frequency multipliers in dc-dc converters (e.g., [11], [12]), this is not usually done, as the output power of a frequency multiplier inverter is inherently low relative to the needed device ratings. However, here we propose using frequency multiplication as an additional operating mode of the inverter and/or rectifier, for widerange voltage and/or power conditions. In this context, frequency multiplication can be used to extend the efficient operating range of a converter and improve its performance across power and voltage.

While the proposed VFX technique can be applied to the inverter and/or rectifier and for wide input and/or output voltage ranges, here we demonstrate it for wide input voltage range using VFX operation of the inverter. Universal input power supplies need to operate over a wide input voltage range and it is a challenge to design resonant power converters for such wide range of operation. In this paper, we demonstrate the VFX technique employed in the inverter of an LLC resonant converter designed to operate across a 4:1 input voltage range of 85 V to 340 V.

This paper is organized as follows. In section II, the variable frequency multiplier technique is introduced and discussed. Section III presents the design and analysis of an LLC converter operating in two VFX modes. Experimental results are presented in section IV, and section V concludes the paper.

### II. VARIABLE FREQUENCY MULTIPLIER TECHNIQUE

The Variable Frequency Multiplier (VFX) technique can be applied to the inverter stage and/or rectifier stage of a converter to achieve wide input voltage and/or output voltage range operation or to extend the efficient operating power range. In this technique, the duty ratio and the switching frequency of an inverter and/or rectifier is changed as input and/or output voltages change such that it processes power between dc and a specific harmonic of its switching frequency (rather than just its fundamental) to create different modes of operation. By operating between dc and a higher harmonic, the dc-ac (or ac-dc) voltage gain of the inverter or rectifier changes, and one gains an added operating mode with different transfer characteristics. In case of frequency control this allows the converter to be operated over a narrower (intermediate ac) frequency range for a wide voltage conversion range and/or power range. Depending on the circuit architecture, more than two modes can be created. To demonstrate the utility of this technique, this paper presents the details of a two mode VFX converter in which the VFX technique is applied to the inverter stage.

To understand the VFX technique applied to an inverter consider the stacked bridge inverter as shown in Fig. 1 with an output voltage  $v_{inv}$  ( $v_{inv} = v_{inv1} + V_{bus} - v_{inv2}$ ). This inverter under VFX control operates in two modes: Fundamental VFX mode (mode 1) and second harmonic VFX mode (mode 2). In mode 1, there are two switching states in one switching period as summarized in Table 1. In state a switches 1 and 4 are on and in state b switches 2 and 3 are on as shown in Fig. 2. Mode 1 results in twice the amplitude of the individual inverter outputs as shown in Fig. 3.

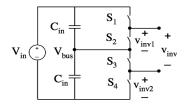


Figure 1. Stacked bridge inverter with input voltage  $V_{in}$  and output voltage  $v_{inv}$ .

TABLE I. SWITCH STATES AND THE VOLTAGE OF THE INVERTER IN FUNDAMENTAL VFX MODE

| State | On Switches | V <sub>inv</sub> |
|-------|-------------|------------------|
| а     | 1, 4        | V <sub>in</sub>  |
| b     | 2, 3        | 0                |

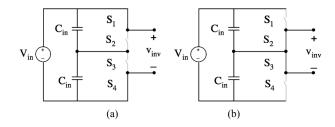


Figure 2. Stacked bridge inverter with input voltage  $V_{in}$  and output voltage  $V_{inv}$  in mode 1; in (a) state a and (b) state b.

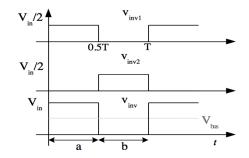


Figure 3. Output voltages of the two inverters  $v_{inv1}$  and  $v_{inv2}$  in Mode 1.

In mode 2, there are four switching states in one switching period as summarized in Table II. The VFX mode results in half the gain and double the frequency of the output waveform for a single switching cycle as shown in Fig. 4. Thus for the transformation stage to see the same frequency as in mode 1, in this mode the converter is operated at half the switching frequency.

 
 TABLE II.
 SWITCH STATES AND OUTPUT VOLTAGE OF THE INVERTER IN THE SECOND HARMONIC VFX MODE

| State | On Switches | V <sub>inv</sub>   |
|-------|-------------|--------------------|
| а     | 1, 3        | V <sub>in</sub> /2 |
| b     | 2, 3        | 0                  |
| с     | 2,4         | V <sub>in</sub> /2 |
| d     | 2.3         | 0                  |

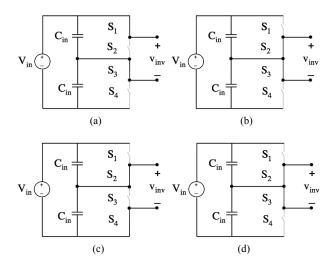


Figure 4. Stacked bridge inverter with input voltage  $V_{in}$  and output voltage  $v_{inv}$  in mode 2; (a) state a and (b) state b, (c) state c, (d) and state d.

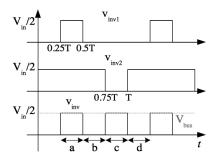


Figure 5. Output voltages of the two inverters  $v_{inv1}$  and  $v_{inv2}$  in Mode 2.

To extend this to other topologies, frequency analysis is useful. Considering Fourier analysis, the square pulse output of each inverter (Fig. 6) can be expressed as the following Fourier series:

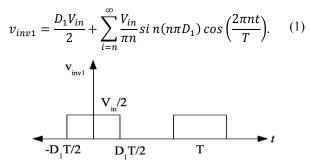


Figure 6. Square pulse train output  $(v_{inv1})$  with duty cycle D<sub>1</sub> and time period T.

Here  $V_{in}$  is the input voltage,  $D_1$  is the duy ratio and T is the time period. Similarly,  $v_{inv2}$  is described by the same equation but with duty cycle  $D_2$ . In mode 1, the duty ratios are identical ( $D_1 = D_2 = 0.5$ ) and  $180^\circ$  out of phase so the fundamental of the half bridge waveforms reinforce. In mode 2,  $D_1 = 0.25$  and  $D_2 = 0.75$  as shown in Fig. 5. In this mode the fundamental of the half bridge waveforms is canceled while the second harmonic is reinforced so the output frequency doubles. Hence, different modes can be created by selecting different duty ratios and time delays between half bridges.

## III. DESIGN OF AN LLC CONVERTER OPERATING WITH VFX CONTROL

We demonstrate the proposed technique in a dc-dc converter designed for a two-stage universal laptop power supply. The ac voltage varies in different countries but the nominal voltage is either 110-120 Vrms at 60Hz, or 220-240 Vrms at 50 Hz. Therefore, 120 V and 240 Vrms have been selected as the upper limits for the two modes of converter operation, corresponding to peak dc voltages of 170 V and 340 V applied to the dc-dc converter. The variable frequency multiplier technique is very useful for this application because there are two distinct peak input voltages.

An LLC converter has been selected for the dc/dc stage. It uses frequency control to regulate the output voltage and has many advantages. The main advantages are that it has the capability to regulate the output voltage over a wide range of input voltage and power with only a small variation in the switching frequency [13]. Also, it achieves zero voltage switching (ZVS) over the entire range of operation thus reducing the switching losses. Moreover the leakage and magnetizing inductance of the transformer can be incorporated into the design.

Figure 7 shows the schematic of the LLC converter with an inverter appropriate for voltage step-down and VFX operation. As it has a high input voltage, stacked half bridges are used. This reduces the voltage stress of the transistors by half, which increases their performance with available devices. The transformation stage consists of a series inductor  $(L_r)$  and a capacitor  $(C_r)$  and a parallel inductor  $(L_m)$ . The capacitor not only provides resonant filtering but also provides dc blocking for flux balancing.

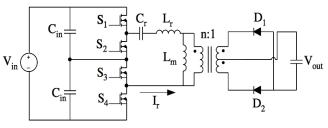


Figure 7. Schematic of the LLC converter with a stacked bridge inverter incorporating the VFX technique.

The transformer parasitics, leakage and magnetizing inductance, can be used instead of separate inductors [14]. A center-tapped transformer is used to reduce the number of series diodes in the rectification path. This increases the loss of the transformer and the voltage stress of the diodes. However, this trade off is still beneficial because of the low output voltage. Synchronous rectification can be used to further reduce losses in the rectification stage [15], [16], [17].

The converter is designed using the method outlined in [18]. Fundamental harmonic analysis (FHA) is used to analyze and design the converter. Time-based [19] and approximate methods [20], [21] can be used for more accurate gain analysis.

The converter is designed for a maximum input voltage of 170 V in the fundamental mode and an output voltage of 20 V. To ensure that the power supply (of which the dc/dc converter is the second stage) has a sufficiently high power factor (i.e., greater than 0.95), the minimum input voltage for the dc-dc stage is 85 V. For input voltages above 170 V, the second harmonic VFX mode is used to decrease the voltage that the transformation stage sees by half.

Using Fundamental Harmonic Analysis, all the voltages and currents are represented by their fundamental components and the secondary-side variables are reflected to the primary side to obtain the approximated circuit shown in Fig. 8.

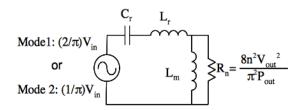


Figure 8. Fundamental harmonic model of the LLC converter.

In mode 1, the dc output voltage of the inverter is  $V_{in}/2$  as shown in Fig. 3. Hence, transformer turns ratio has been selected as:

$$n = \frac{V_{in-max}}{2V_{out}} = 4.25.$$
 (2)

The recommended range of the ratio of  $L_m/L_r$  (referred to as k) is between 3 to 10 [18]. Smaller values of k result in a narrow and steep gain curve but a much higher magnetizing current, resulting in higher loss. To have a reasonable minimum frequency, magnetizing current and dead time, the value of k is chosen as 7. The value of k can be optimized for a narrower frequency range or a higher efficiency depending on the intended application. The maximum gain ( $M_{max}$ ) is selected higher than 2 (i.e, 2.4) to ensure sufficient gain even with the inaccuracies of using fundamental harmonic analysis.

To ensure ZVS, the input impedance of the resonant network needs to be inductive at the drive frequency. The borderline between the inductive and capacitive region is when the impedance is purely resistive. By equating the imaginary part of input impedance  $(x - \frac{1}{x} + \frac{xk}{1+k^2x^2Q^2})$  equal to zero the value of Q is found  $(Q = \sqrt{\frac{1}{(1-x^2)k} - \frac{1}{k^2x^2}})$ . Substituting the value of Q in the expression for gain  $(M = \sqrt{1/(1 + \frac{1}{k}(1 - \frac{1}{x^2})^2 + Q^2(x - \frac{1}{x})^2})$  we can get the maximum gain  $(M_{max} = x/\sqrt{x^2(1 + \frac{1}{k}) - \frac{1}{k}})$ . At the maximum gain we get the minimum normalized frequency. This value of  $x_{min}$  is substituted in the expression of Q to get the maximum Q below which ZVS is maintained.

$$Q_{max} = \frac{1}{k} \sqrt{\frac{1 + k(1 - \frac{1}{M_{max}^2})}{M_{max}^2 - 1}} = 0.1706.$$
 (3)

The values of  $n, k, R_n$  and  $Q_{max}$  are used to calculate  $L_r, L_m$  and  $C_r$ :

The dead time should be sufficient such that the current in the inductor  $L_m$  at the switching instant can discharge the voltage on the MOSFET before it is switched on. By equating the charge required to the current in inductor  $L_m$ 

$$L_{r} = \frac{Q_{max} R_{n}}{w_{r}} = 6.36 \,\mu\text{H},$$

$$L_{m} = k L_{r} = 44.5 \,\mu\text{H},$$

$$C_{r} = \frac{1}{Q_{max} R_{n} w_{r}} = 15.9 \,\mu\text{F}.$$
(4)

during the dead time, the deatime is calculated as  $t_d = 8C_{ds}f_rL_m = 62$  ns. Using Fundamental Harmonic Analysis, the gain curve of the transformation stage is given in Fig. 9. If the converter had not been designed considering the VFX technique, double the gain and transformer turns ratio would be needed to operate over the entire range, thus increasing the total loss.

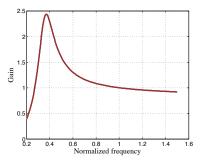


Figure 9. The gain of the transformation stage with peak gain at 0.4 times the normalized resonant tank  $(L_r \text{ and } C_r)$  frequency.

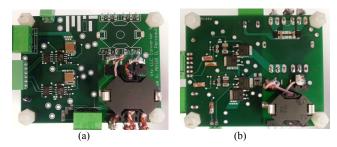


Figure 10. Picture of the (a) top side and (b) bottom side of the prototype board.

#### IV. EXPERIMENTAL PROTOTYPE AND RESULTS

Using the design values from the previous section, a prototype for the converter was built as seen in Fig. 10. The components used for the experimental prototype are summarized in Table III.

TABLE III. COMPENENTS USED IN THE EXPERIMENTAL PROTOTYPE

| Components   | Туре   |
|--------------|--|
| Controller   | 150 MHz digital signal controller (TI's  |
|              | TMS320F28335)  |
| Signal       | 150 Mbps two channel digital isolator (NVE   |
| Isolators    | Corporation's IL711), Qty: 2   |
| Gate Drivers | 600-V/4-A High and low side gate driver  |
|              | (IRS21867S), Qty: 2  |
| Transistors  | 200-V/34-A OptiMos power transistor (Infineon's  |
|              | IPD320B20N3), Qty: 4   |
| Capacitors   | C <sub>r</sub> : 15.99 pF/250 V, C <sub>out</sub> : 20 µF/25-V, C <sub>in</sub> : 1 µF/250-V |
|              | Qty: 2   |
| Inductor     | L <sub>r</sub> : 3.6 µH, RM8A100 3F3 core, litz wire (6 turns, 48                            |
|              | AWG, 1000 strands).  |

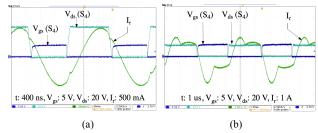
| Transformer | RM10A160 3F3 core, Primary litz wire (17 turns, 46 AWG, 450 strands). Secondary litz wire (4 turns, 46 |  |
|-------------|--|--|
|             | AWG, 450 strands).   |  |
| Diodes      | 60-V/3-A Schottky diode (NXP's PMEG6030EVP)  |  |

The dc/dc converter is a step-down converter operating at the series tank  $(L_r, C_r)$  resonant switching frequency  $(f_r)$ of 500 kHz. It has an input voltage range of 85 V - 340 V, fixed output voltage of 20 V and a rated output power of 50 W. Table IV summaries the converter specifications. To control the output power and gain, frequency control is utilized, while the appropriate VFX mode is used based on input voltage being above or below 170 V. The transformer was designed to exploit the integrated magnetizing inductance. The leakage inductance was used as part of the resonant inductance. However, this was insufficient and a series inductor was added to provide the required series resonant inductance.

TABLE IV. PROTOTYPE CONVERTER SPECIFICATIONS

| Parameter                              | Value        |
|--|--------------|
| Input voltage (V <sub>in</sub> )       | 85 V – 340 V |
| Output voltage (Vout)                  | 20 V         |
| Output power (P <sub>out</sub> )       | 5W – 50 W    |
| Switching frequency (f <sub>sw</sub> ) | 500 kHz      |

The prototype was tested using a resistive load. The converter operates with ZVS across the entire range of operation. The switching waveforms for input voltages of 170 V and 85 V at 50 W in mode 1 are given in Fig. 11 (a) and (b), respectively. It shows the current input to the transformer primary which is also the output current of the bottom inverter, the gate drive voltage of switch  $S_4$  and the drain-source voltage of switch S<sub>4</sub>. At 170 V (Fig. 11 (a)) the current is sinusoidal with a cusp at the switching instants. The converter was operated below resonance, to increase the gain of the transformation stage, as the input voltage decreased. As the converter is operated away from resonance the current waveform distorts and does not remain sinusoidal. However, the experimental gain is very similar to that calculated by fundamental harmonic analysis and ZVS is still maintained.



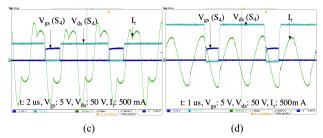
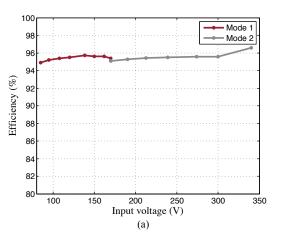


Figure 11. Current and voltage waveforms of the converter at 50 W when operated in mode 1, fundamental mode, at input voltage of (a) 170 V and (b) 85 V and when operated in mode 2, VFX mode, at input voltage of (c) 170 V and (d) 340 V. (1-Blue) Gate voltage of switch S<sub>4</sub>, (2-Turquoise) drain-source voltage of switch S<sub>4</sub>, and (4-Green) current output of lower half-bridge that is flowing in to the transformer primary

For input voltages above 170 V, operation is changed to the VFX mode and the waveforms for 170 V and 340 V are given in Fig. 11 (c) and (d), respectively. The converter is operated at half the normal-mode switching frequency, which decreases the frequency-dependent switching losses and ZVS is still maintained resulting in high efficiency.

The efficiency of the converter was measured across input voltage in both modes and across output power. The measured efficiency of the converter at rated power as a function of input voltage is plotted in Fig. 12 (a). The converter continues to operate with high efficiency with the mode change and the converter efficiency varies from 94.9% to 96.6% across the entire range of input voltages. The measured efficiency as a function of output power and fixed input voltage of 170 V in fundamental mode varies from 86% to 95.4% and is plotted in Fig. 12 (b). The high efficiency over a wide operation range demonstrates the effectiveness of the VFX technique.



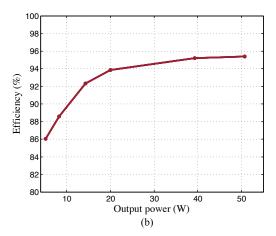


Figure 12. Efficiency of the converter with variation in (a) input voltage with fixed output voltage and 50 W output power and with variation in (b) output power with 170 V input voltage and fixed output voltage.

#### V. CONCLUSION

This paper presents a Variable Frequency Multiplier (VFX) technique for resonant power converters. The approach is applied to the inverter of an LLC converter, to demonstrate the effectiveness of this technique for universal input power supplies. This technique increases the input voltage range by a factor of two and the converter achieves high efficiency over a wide range of operation. The experimental prototype is able to achieve an efficiency of 94.9% to 96.6% across the entire input voltage range at 50 W output power and 86% to 95.4% across a 10:1 power range with 170 V input voltage. Hence, the VFX technique can be very useful to obtain high efficiency across a wide range of operation.

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