

# Design and Control of Lossless Multi-Way Power Combining and Outphasing Systems

Alexander S. Jurkov and David J. Perreault

Laboratory for Electric and Electromagnetic Systems  
Massachusetts Institute of Technology  
Cambridge, MA, USA

Invited Paper

**Abstract**—A lossless multi-way power combining and outphasing system has been recently proposed which offers major performance advantages over conventional approaches such as Chireix power combining. This paper presents a new outphasing control strategy for the proposed system that enables output power control through effective load modulation of the power amplifiers while minimizing susceptive variations in loading. Moreover, a simple methodology for designing the combiner is introduced.

## I. INTRODUCTION

Radio-frequency (RF) power amplifiers find wide applicability in numerous areas including RF communications, industrial processing, and power conversion. Such power amplifiers (PAs) are often constrained by two requirements: (1) the ability to provide linear and dynamic control of their output power over a wide range, and (2) the necessity to maintain high efficiency across the operating power range.

One approach for simultaneously satisfying both of these constraints, proposed originally in the 1930's, is the method of *outphasing* [1]. Traditionally, this method (see Fig. 1) consists of decomposing the desired input signal to be amplified  $S_{in}(t)$  into two constant-amplitude signals  $S_1(t)$  and  $S_2(t)$ . These appropriately phase-shifted (outphased) signals are amplified and combined (summed) to yield an amplified version  $S_{out}(t)$  of the input signal [2]. The fact that  $S_1$  and  $S_2$  have constant amplitudes enables the use of highly-efficient switched-mode PAs [3, 4].

One traditional implementation of the outphasing concept is the Chireix combiner (Fig. 2A) with two input power ports  $P_{in,A}$  and  $P_{in,B}$ , and one output power port  $P_{out}$  [1]. The PAs driving the input ports must be appropriately outphased for the desired output power to be delivered [1]. Although this combiner is ideally lossless, its reactive loading on the PAs varies substantially with outphasing (and power delivery) and is only zero for at most two output power levels. Variations in the reactive loading of the amplifiers can adversely affect their performance and give rise to reactive currents that limit the efficiency of the overall system [5].

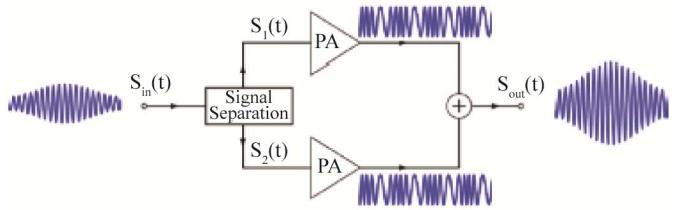


Figure 1. The traditional outphasing method. A desired output envelope is created by appropriately outphasing two constant-envelope signals [1,2,5,6].

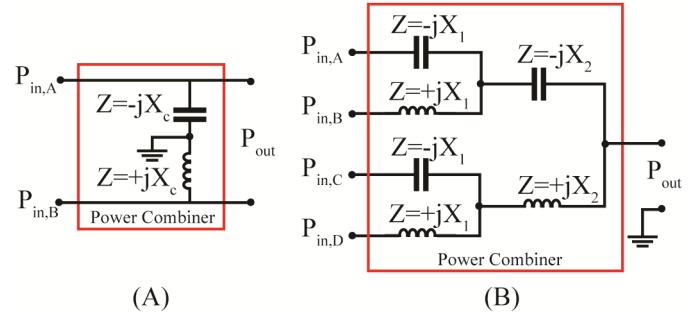


Figure 2. (A) Chireix combiner; (B) A four-way implementation of the proposed multi-way combiner.

Recently, a new power combining and outphasing system has been introduced that overcomes the loss and reactive loading problems of traditional outphasing approaches such as the Chireix combiner [6,7]. It provides ideally lossless power combining from four or more PAs, along with nearly resistive loading of the individual power amplifiers over a very wide output power range. One possible implementation of this combiner for four PAs (four-way combiner) is shown in Fig. 2B. It has been theoretically shown that the proposed combiner considerably improves system efficiency [7].

The purpose of this paper is to introduce a new outphasing control strategy for the proposed combining system which allows the control of output power while minimizing susceptive variations in the loading of the power amplifiers

over a specified operating power range. Further, the paper presents a straightforward method for designing the combiner.

## II. OVERVIEW OF THE NEW COMBINER

Consider the four-way combiner of Fig. 3 driving a resistive load  $R_L$ . For modeling purposes, the PAs have been treated as ideal sinusoidal voltage sources  $V_A - V_D$  with constant amplitude  $V_S$  and phasor relationships according to Fig. 4.

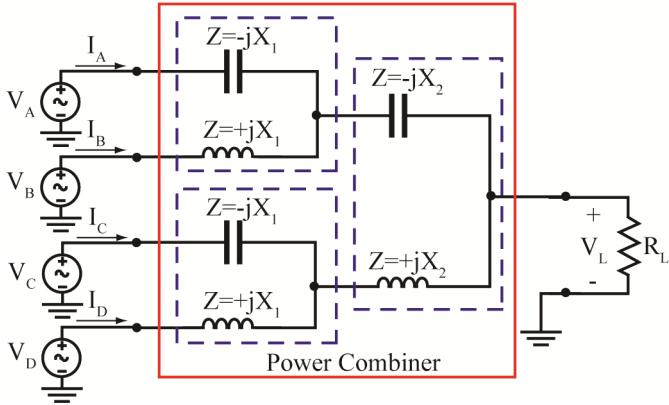


Figure 3. Four-way combiner driving a resistive load, with PAs treated as ideal sinusoidal voltage sources [6,7]. The four-way combiner can be thought as an interacting cascade of two-way combining elements.

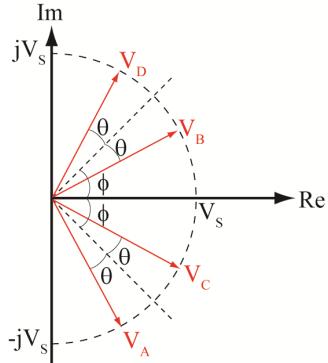


Figure 4. Proposed phasor relationship among voltage sources  $V_A - V_D$  driving the four-way combiner of Fig. 3 [6,7].

One can view the four-way combiner as an interacting cascade of two-way combining structures (in which the details of the interactions are important to the overall system operation). Note that this approach can be extended to synthesizing higher-level combiners. For example, an eight-way combiner can be obtained by driving each of the inputs of a four-way structure with an added two-way structure. Although this paper will address only the four-way combiner, the presented concepts can easily be adapted for the general case of a  $2^N$ -way combiner.

### A. Power Amplifier Loading Characteristics

It is of particular interest to know the effective admittances seen by the PAs driving the combiner for a given set of control angles  $[\theta, \phi]$ . The effective admittance at a combiner input port is the complex ratio of current to voltage at the port with all driving sources active. It can be shown that for the four-

way combiner of Fig. 3 driven according to the phase relationship in Fig. 4, the effective admittance at each input port A-D is respectively given by (1)-(4), where  $\gamma = R_L/X_1$  and  $\beta = X_2/X_1$  [6,7].

$$Y_{eff,A} = X_1^{-1}(\gamma - \gamma \cos(2\phi + 2\theta) - \gamma \cos(2\phi) + \gamma \cos(2\theta) - \beta \sin(2\phi)) \quad (1)$$

$$+ jX_1^{-1}(1 - \beta - \gamma \sin(2\phi + 2\theta) - \gamma \sin(2\phi) + \gamma \sin(2\theta) + \beta \cos(2\phi))$$

$$Y_{eff,B} = X_1^{-1}(\gamma - \gamma \cos(2\theta - 2\phi) - \gamma \cos(2\phi) + \gamma \cos(2\theta) + \beta \sin(2\phi)) \quad (2)$$

$$+ jX_1^{-1}(-1 - \beta - \gamma \sin(2\theta - 2\phi) + \gamma \sin(2\phi) + \gamma \sin(2\theta) + \beta \cos(2\phi))$$

$$Y_{eff,C} = X_1^{-1}(\gamma - \gamma \cos(2\theta - 2\phi) - \gamma \cos(2\phi) + \gamma \cos(2\theta) + \beta \sin(2\phi)) \quad (3)$$

$$- jX_1^{-1}(-1 - \beta - \gamma \sin(2\theta - 2\phi) + \gamma \sin(2\phi) + \gamma \sin(2\theta) + \beta \cos(2\phi))$$

$$Y_{eff,D} = X_1^{-1}(\gamma - \gamma \cos(2\phi + 2\theta) - \gamma \cos(2\phi) + \gamma \cos(2\theta) - \beta \sin(2\phi)) \quad (4)$$

$$- jX_1^{-1}(1 - \beta - \gamma \sin(2\phi + 2\theta) - \gamma \sin(2\phi) + \gamma \sin(2\theta) + \beta \cos(2\phi))$$

It can be seen from (1)-(4) that the input admittances seen at ports A and D are complex conjugates, as are those seen at ports B and C.

### B. Output Power Characteristics

Understanding of the relationship between the outphasing angles  $\theta, \phi$  and the output power delivered to a resistive load  $R_L$  is useful when considering various outphasing control strategies. By employing straightforward linear circuit analysis techniques, it can be shown that the output power is concisely expressed by (5) for any pair of outphasing control angles  $[\theta, \phi]$ .

$$P_{out} = \frac{8R_L V_S^2}{X_1^2} \sin^2(\phi) \cos^2(\theta) \quad (5)$$

Moreover, it can be readily seen from (5) that the maximum output power deliverable by the combiner, the saturated output power  $P_{out,sat}$ , is given by (6), and corresponds to  $\theta = 0^\circ$  and  $\phi = 90^\circ$ .

$$P_{out,sat} = \frac{8R_L V_S^2}{X_1^2} \quad (6)$$

## III. OUTPHASING CONTROL

As can be observed from (5), the output power depends on both  $\theta$  and  $\phi$ , i.e. there are two degrees of freedom and only one equation. Thus, an additional constraint on  $\theta$  and  $\phi$  can be specified. The nature of this constraint is determined by the combiner operating characteristics one desires to attain (for example, the behavior of the combiner input admittances over the output power range). This section briefly overviews the original control strategy employed in [6, 7] and presents a new outphasing control law which minimizes the susceptive components of the input admittances over the entire operating range.

### A. Overview of the Original Control Strategy

The outphasing control strategy employed in the original work on the proposed combiner [6,7] results from the approximate inverse operation of a resistance compression network (RCN) such as the one explored in [8]. The inverse operation of an RCN is one possible method for synthesizing the new combiner, and is rigorously described in [7]. Here we

only present the Approximate Inverse RCN (AIRCN) outphasing strategy for comparison purposes. According to AIRCN, the outphasing angles are selected as [6, 7]:

$$\theta = \text{ATAN} \left( \frac{4V_s^2 X_2 P_{cmd}}{4V_s^4 + X_1^2 P_{cmd}^2} \right) \quad (7)$$

$$\phi = \text{ATAN} \left( \frac{X_1 P_{cmd}}{2V_s^2} \right) \quad (8)$$

where  $P_{cmd}$  is the "commanded" output power and can be regarded as the desired output power (for a system with linear output power control, it is equal to the actual output power). AIRCN control yields simple analytical expressions for  $\theta$  and  $\phi$ , and results in small susceptive components in the input admittances (although not minimal or equal). Fig. 5 depicts the input susceptive components seen by the PAs driving the combiner of Fig. 3 for an example design with  $R_L = 50 \Omega$ ,  $X_1 = 35.60 \Omega$ , and  $X_2 = 48.78 \Omega$ . As can be observed, there are exactly four output power levels (*zero-points*) for which the input susceptances are zero - an important characteristic of the combiner. Fig. 6 illustrates the relationship between the actual output power  $P_{out}$  and the "commanded" power  $P_{cmd}$ . Note that  $P_{out}$  does not precisely track  $P_{cmd}$  with the AIRCN control strategy. However, this nonlinearity can be readily addressed through predistortion or appropriate (compensating) adjustment of the control law.

### B. Optimal Susceptance Control

The presently proposed optimal susceptance (OS) control strategy is characterized with three main advantages over AIRCN control: (1) it minimizes the susceptance seen by the PAs for any output power level, (2) it achieves equal (magnitude) susceptive loading of the PAs over the operating range, and (3) provides linear control of output power. For the four-way combiner addressed here (see Fig. 3), the OS control angle pair  $[\theta, \phi]$  can be computed for a particular output power level  $P_{out}$  by numerically minimizing the largest susceptive component magnitude in the input admittances seen among the PAs (1)-(4) at  $P_{out}$  subject to the constraint (5). It can be shown that for the output power range given by:

$$\frac{P_{out,sat}}{2} \left( 1 - \sqrt{1 - \frac{X_1^2}{4R_L^2}} \right) \leq P_{out} \leq \frac{P_{out,sat}}{2} \left( 1 + \sqrt{1 - \frac{X_1^2}{4R_L^2}} \right) \quad (9)$$

which extends beyond the range of the zero-points (covering most of the range of practical interest), the solutions of the preceding optimization problem reduce to a set of convenient analytical expressions for calculating the control angles:

$$\theta = \text{ACOS} \left( \sqrt{\frac{4V_s^4 + P_{out}^2 X_1^2}{8P_{out} R_L V_s^2}} \right) \quad (10)$$

$$\phi = \text{ATAN} \left( \frac{X_1 P_{out}}{2V_s^2} \right) \quad (11)$$

Fig. 5 shows the input susceptances seen by the PAs driving the combiner of Fig. 3 with  $R_L = 50 \Omega$ ,  $X_1 = 35.60 \Omega$ ,  $X_2 = 48.78 \Omega$ . Compared to AIRCN control, the OS strategy

reduces and evenly distributes the worst-case effective susceptance among the PAs for any particular output power level. As expected, the zero-points coincide with those of the AIRCN control. Moreover, the OS control guarantees that the "commanded" power is equivalent to the actual output power provided that the commanded power level is below the saturation power (approximately 0.31 W for the considered example with drive voltage amplitudes of  $V_s = 1 \text{ V}$ ). (See Fig. 6.)

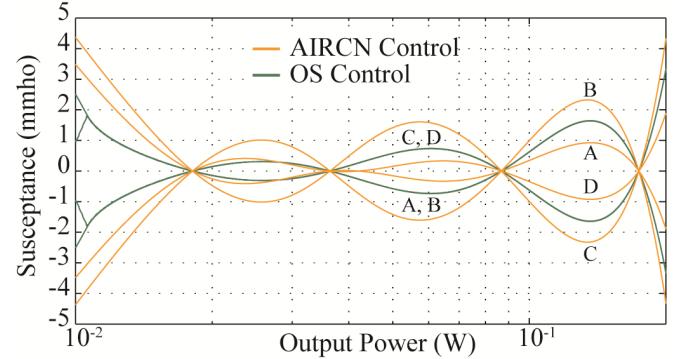


Figure 5. Effective susceptive component of admittance seen by PAs A-D driving the four-way combiner of Fig. 3 with  $R_L = 50 \Omega$ ,  $X_1 = 35.60 \Omega$ ,  $X_2 = 48.78 \Omega$  for both the AIRCN and OS control strategies.

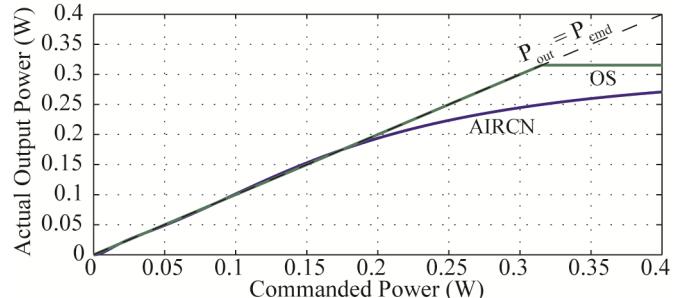


Figure 6. Actual output power versus "commanded" power for the example system ( $V_s = 1 \text{ V}$ ,  $R_L = 50 \Omega$ ,  $X_1 = 35.60 \Omega$ ,  $X_2 = 48.78 \Omega$ ) for AIRCN and OS control.

## IV. COMBINER DESIGN

Although we have so far described an outphasing method for controlling the combiner, an important question remains unanswered – how to best select the combiner component values. This section briefly presents a straightforward approach for determining the reactances  $X_1$  and  $X_2$  of the four-way combiner (Fig. 3) for a particular output power operating range specification.

The reactance magnitude  $X_2$  is chosen to be slightly less than the load resistance  $R_L$ , as given by (12). The design parameter  $k$ , equal or greater than 1, is selected depending on the particular performance specifications on the combiner.

$$X_2 = \frac{2R_L}{k+1} \quad (12)$$

The reactance magnitude  $X_1$  is then computed according to:

$$X_1 = \frac{X_2}{k + \sqrt{k^2 - 1}} \quad (13)$$

This approach originates from the design of multi-stage resistance compression networks [7]. The performance and behavior of each power combiner with reactances selected as outlined above are uniquely determined by the particular  $k$  value. Fig. 7 depicts an absolute value of the maximum effective input susceptance of the four-way combiner of Fig. 3 for two example designs with  $k = 1.05$  and  $k = 1.25$ . It can be demonstrated that in general (as can be inferred from Fig. 7), smaller  $k$  values result in narrower power operating ranges associated with smaller worst-case input susceptance, while larger  $k$  values allow wider operating ranges at the expense of higher worst-case susceptance magnitude.

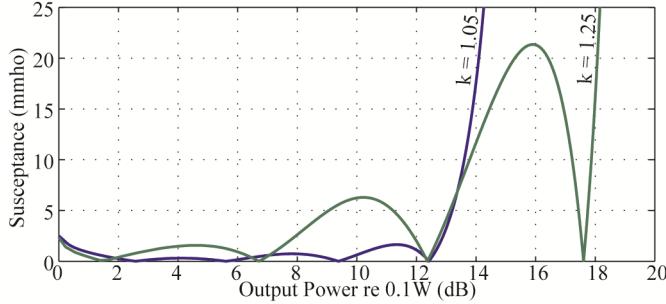


Figure 7. Worst-case effective input susceptance magnitude versus output power for the four-way combiner of Fig. 3 ( $V_s = 1$  V and  $R_L = 50 \Omega$ ) with OS control for  $k = 1.05$  and  $k = 1.25$ .

Fig. 8 provides a set of numerically-computed design curves which facilitate the selection of the optimal  $k$  value for a particular output power range ratio (PRR) for each of AIRCN and OS control. The PRR is the ratio of the maximum to the minimum output power over which the susceptance is to be minimized. The obtained  $k$  value is optimal in the sense that it results in the smallest worst-case susceptance magnitude over the specified operating PRR. The value of  $k$  is found by tracing horizontally from the specified PRR to the Power Ratio Curve of interest, and then vertically to the  $k$ -axis. The corresponding worst-case loading susceptance can be obtained by first tracing the selected  $k$  to the appropriate Susceptance Curve, and then horizontally to the Susceptance axis.

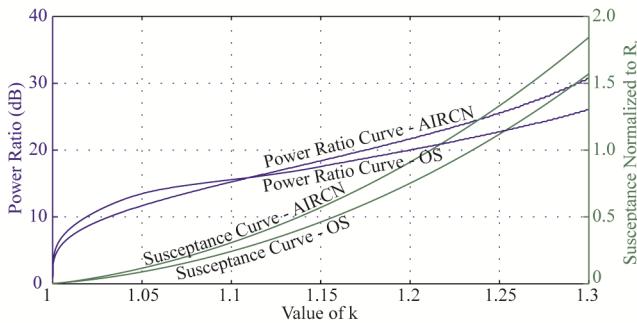


Figure 8. Input susceptance-minimizing design curves for a four-way combiner for the AIRCN and OS outphasing control methods. To de-normalize susceptance axis, simply divide by  $R_L$ .

Fig. 9 shows the actual minimum and maximum output power levels over which this minimum PA loading susceptance is achieved for each value of  $k$ . The power axis can be de-normalized by multiplying by  $V_s^2 / R_L$ . Note that the operating range defined by the minimum and maximum output power curves is simply one over which there is a particular interest in minimizing the input susceptance.

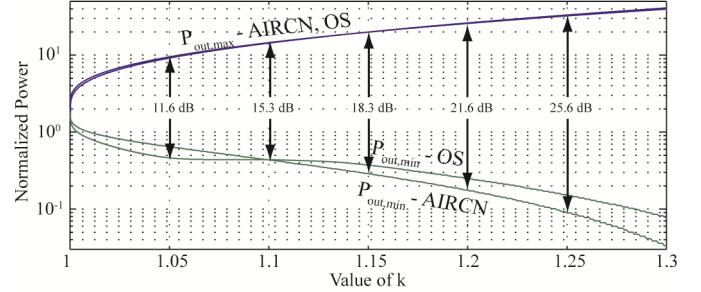


Figure 9. Normalized limits of the output power operating range corresponding to a particular value of  $k$ . De-normalize the power axis by multiplying by  $V_s^2 / R_L$ .

## V. CONCLUSION

This paper presented a brief overview of a recently proposed lossless multi-way outphasing system that offers major performance advantages over conventional outphasing and combining approaches such as Chireix. A new optimal susceptance outphasing control strategy is introduced that allows output power control through effective load modulation of the power amplifiers while minimizing susceptive variations in loading. In addition, a simple methodology is described for designing the combiner to meet a set of performance specifications.

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